Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

A2 Mathematics (9MA0)

Paper 02 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks	AOs
1(a)	$\{f'(x)=\}\ldots x^2+\ldots x+\ldots \Rightarrow \{f''(x)=\}\ldots x+\ldots$	**M1**	1.1b
	$\{f'(x)=\}3x^2+4x-8 \Rightarrow \{f''(x)=\}6x+4$	**A1cso**	1.1b
		(2)	
(b)(i)	$"6x+4"=0 \Rightarrow x="-\frac{2}{3}"$	**B1ft**	1.1b
(ii)	$x \leqslant "-\frac{2}{3}"$ or $x<"-\frac{2}{3}"$	**B1ft**	2.2a
		(2)	
			(4 marks)

Notes

(a)

M1: For attempting to differentiate twice.

It can be scored for any of: $x^3 \to \ldots x^2 \to \ldots. x$ or $2x^2 \to \ldots x \to k$ or $-8x \to k \to 0$ where … are constants.

You can ignore the lhs so do not be concerned what they call the first and/or second derivative, just look for their expressions.

The indices do not need to be processed for this mark so allow for e.g. $x^3 \to \ldots x^{3-1} \to \ldots. x^{3-1-1}$

A1cso: $(f''(x)=)\ 6x+4$ Correct second derivative from fully correct work. The $"f''(x)="$ is not required.

Allow $6x^1$ for $6x$ but not $4x^0$ for 4 unless the $4x^0$ becomes 4 later, e.g. in part (b).

Do **not** apply isw so mark their final answer. E.g. if $6x + 4$ becomes $3x + 2$ score A0

(b)

(i)

B1ft: $ax+b=0 \Rightarrow (x=)-\frac{b}{a}$. This mark is for obtaining $x=-\frac{2}{3}$ **or** $x=-\frac{b}{a}$ which has come from solving an equation of the form $ax+b,\ a,b \neq 0$ where $ax + b$ is their attempt to differentiate twice in part (a)

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$ or an exact decimal and isw.

(ii)

B1ft: Deduces $x \leqslant -\frac{2}{3}$ **or** follow through their single value of x from part (i) obtained from their attempt to solve an equation of the form $ax+b=0,\ a,b \neq 0$ where $ax + b$ was their attempt to differentiate twice in part (a) . Do not isw and mark their final answer.

If 2 inequalities are given e.g. $x<"-\frac{2}{3}",\ x>"-\frac{2}{3}"$ without indicating which is their answer score B0

Condone $<$ for $\leqslant$ and allow equivalent inequalities e.g. $-\frac{2}{3}>x$

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$

Allow equivalent notation so these are all acceptable:

$x \leqslant "-\frac{2}{3}",\ x<"-\frac{2}{3}",\ \left(-\infty,"-\frac{2}{3}"\right],\ \left(-\infty,"-\frac{2}{3}"\right),\ \left\{x : x \leqslant "-\frac{2}{3}"\right\},\ \left\{x : x<"-\frac{2}{3}"\right\}$

Ignore any reference to values of y.

Allow ft decimal answers from (i) which may be inexact.

Correct answers in part (b) with no working in (a) can score 0011.

Question	Scheme	Marks	AOs
2(a)(i)	e.g. $(u_2=)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40$ *	B1*	2.1
(ii)	$u_3=40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2\ (=28)$ **or** $u_4="28"+7\cos\left(\frac{3\pi}{2}\right)-5(-1)^3\ (=33)$	M1	1.1b
	$u_3=28$ and $u_4=33$	A1	1.1b
		(3)	
(b)(i)	$(u_5=)35$	B1	2.2a
(ii)	e.g. $\sum_{r=1}^{25}u_r=6(35+40+"28"+"33")+35$	M1	3.1a
	= 851	A1	1.1b
		(3)	
		(6 marks)	

Notes

(a)
(i)
B1*: Correct application of the formula with n = 1 and proceeds correctly to achieve an answer of 40 with no errors. Note that e.g., $(u_2=)35+7\cos\left(\frac{35\pi}{2}\right)-5(-1)^{35}=35+0+5=40$ scores B0

As a minimum need to see e.g. $(u_2=)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40,\quad 35+0+5=40,\quad 35+5=40,\quad 35-5(-1)^1=40$

(ii)
M1: A correct attempt to use the formula to find a value for u_3 or u_4
Look for n = 2 substituted correctly into the given formula with $u_2=40$. May be implied by $u_3=28$
Or their calculated value of u_3 used with n = 3 substituted correctly into the given formula to find u_4
Condone use of calculator in degree mode which gives $u_3=41.989...$ which may imply this mark if no working is shown. If there is **no** working and u_3 is incorrect and u_4 is correct score M0A0
A1: Both correct $u_3=28$ and $u_4=33$ If 28, 33 are listed then allow M1A1.
For both correct values only score M1A1

(b)(i)
B1: $(u_5=)35$
(ii)
M1: Attempts a **<u>correct</u>** method to find $\sum_{r=1}^{25}u_r$
There are various ways e.g. attempts to add 35 to $6\times$ the sum of their four values.
Some other examples are:
$\sum_{r=1}^{25}u_r=7\times35+6\times40+6\times"28"+6\times"33"$, $\sum_{r=1}^{25}u_r=7(35+40+"28"+"33")-(40+"28"+"33")$,
$\sum_{r=1}^{25}u_r=\frac{25}{4}(35+40+"28"+"33")+1$, $2(35+40+"28"+"33")=272$, $272\times3=816$, $816+35$
There may be other methods seen but the calculation must be correct for their values.
If there is no working, with incorrect u_3 and/or u_4 you will need to check if their answer implies a correct method using $6(35+40+"28"+"33")+35$
Attempts to use an AP/GP formula score M0
A1: 851 (Correct answer with no working scores both marks)

Question	Scheme	Marks	AOs
3(a)	Uses one correct log law e.g. $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$ $2=\log_2 4,\ \ 2\log_2 x=\log_2 x^2$	**M1**	1.1b
	$(x+3)(x+10)=4x^2$ oe	**dM1**	2.1
	$\Rightarrow 3x^2-13x-30=0$ *	**A1***	1.1b
		(3)	
(b)(i)	$(x=)\ 6,\ -\frac{5}{3}$	**B1**	1.1b
(ii)	$x\neq-\frac{5}{3}$ because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real	**B1**	2.4
		(2)	
			(5 marks)

Notes

(a)

M1: Uses one correct log law. The base does not need to be seen for this mark.
This mark is independent of any other errors they make.
Examples: $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$, $2=\log_2 4$, $2\log_2 x=\log_2 x^2$

dM1: Fully correct work leading to a correct equation not containing logs that is not the printed answer. **Depends on the first mark.** Condone a spurious base e.g. 10 or e, so long as the log work is otherwise correct (i.e., they recover the base 2).
Examples (depending on their method): $(x+3)(x+10)=4x^2$, $\frac{(x+3)(x+10)}{x^2}=4$, $\frac{x+3}{x^2}=\frac{4}{x+10}$
or $1+\frac{13}{x}+\frac{30}{x^2}=4$
Allow recovery from invisible brackets but **not** from incorrect work e.g.

$$\log_2(x+3)+\log_2(x+10)=2+2\log_2 x \Rightarrow \log_2(x+3)(x+10)-\log_2 x^2=\log_2 4$$

$$\Rightarrow \frac{\log_2(x+3)(x+10)}{\log_2 x^2}=\log_2 4 \Rightarrow \frac{(x+3)(x+10)}{x^2}=4$$

This scores M1dM0A0

A1*: Obtains $3x^2-13x-30=0$ with no processing errors but condone a spurious base e.g. 10 or e, so long as the log work is otherwise correct (i.e., they recover the base 2) and allow recovery from invisible brackets.

Note the following alternative which can follow the main scheme:

$\log_2(x+3)+\log_2(x+10)=2+2\log_2 x=2+\log_2 x^2$ **M1**

$2^{\log_2(x+3)+\log_2(x+10)}=2^{2+\log_2 x^2} \Rightarrow 2^{\log_2(x+3)}\times 2^{\log_2(x+10)}=2^2\times 2^{\log_2 x^2} \Rightarrow (x+3)(x+10)=4x^2$ **dM1**

$\Rightarrow 3x^2-13x-30=0$ * **A1**

Special Cases:

1. $(x+3)(x+10)=4x^2$ with no working leading to the correct answer scores **M1dM1A0**

2. $\log_2(x+3)+\log_2(x+10)=2+2\log_2 x \Rightarrow 2^{\log_2(x+3)+\log_2(x+10)}=2^{2+2\log_2 x} \Rightarrow (x+3)(x+10)=4x^2$

$\Rightarrow 3x^2-13x-30=0$ *

Also scores **M1(implied)dM1A0** (lack of working)

(b)(i)

B1: Both values correct: $(x =)\ 6,\ -\dfrac{5}{3}$

(b)(ii)

B1: e.g. $(x \neq) -\dfrac{5}{3}$ and $\log_{(2)}\left(-\dfrac{5}{3}\right)$ is not real

This mark requires the identification of the **correct** negative root **and** an acceptable explanation.

For the identification of the root allow e.g. $x \neq -\dfrac{5}{3},\ x = -\dfrac{5}{3},\ -\dfrac{5}{3}$ etc. as long as it is clear they have identified the correct value. Requires the correct negative root $\left(-\dfrac{5}{3}\right)$ but the other may not be 6 but must be positive.

Some examples for the explanation:

- you get $\log_{(2)}\left(-\dfrac{5}{3}\right)$ which is not possible
- $\log -\dfrac{5}{3}$ is not possible, can't be found, gives a math error, is not real, is undefined
- if $\left\{k = \log_2\left(-\dfrac{5}{3}\right),\right\}\ 2^k = -\dfrac{5}{3}$ which is not possible
- you get log of a negative number
- negative numbers can't be "logged"
- log of negative does not work

Do not allow e.g.

- you can't have a negative log, logs can't be negative (unless clarified further)
- "you get a math error" in isolation
- a log cannot have a negative value
- logs cannot be negative
- $-\dfrac{5}{3}$ is not a valid input (unless clarified further)
- "it doesn't work in the logs"
- log graph isn't negative
- log graph does not cross negative x–axis
- x is only positive & negative answer does not work

Allow an implied correct answer if they say e.g. 6 is the root because $\log_{(2)}\left(-\dfrac{5}{3}\right)$ is not possible

Question	Scheme	Marks	AOs
4(a)	$(A=)\,55$	B1	3.4
		(1)	
(b)	$\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}-ABe^{-Bt}$ or $\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}-\text{"55"}Be^{-Bt}$	M1	3.1b
	$-B\times\text{"55"}=-7.5\Rightarrow B=...\ \left(\dfrac{3}{22}=\text{awrt } 0.136\right)$	M1	1.1b
	$H=55e^{-0.136t}+30$	A1cso	3.3
		(3)	
			(4 marks)

Notes

(a)

B1: 55 only. Just look for this value e.g. "A =" is not required. Ignore any "units" if given e.g. 55 °C

(b)

M1: Differentiates to obtain an expression of the form $\pm ABe^{-Bt}$ which may have their A already substituted in so allow for $\pm ABe^{-Bt}$ or $\pm\text{"55"}Be^{-Bt}$

M1: Substitutes t = 0 and their A into their $\dfrac{\mathrm{d}H}{\mathrm{d}t}$, sets $=\pm 7.5$ and proceeds to find a value for B which may be implied by $\dfrac{3}{22}$ or awrt 0.136

Their $\dfrac{\mathrm{d}H}{\mathrm{d}t}$ must not be H. i.e. it must be a "changed" function.

A1cso: Correct **equation** which follows **fully correct work** $H=55e^{-0.136t}+30$ but condone $H=55e^{-\frac{3}{22}t}+30$ The final equation must be correct but you can ignore spurious notation within their solution such as integral signs and "+ c" which do not affect their solution.

<u>Marking guidance is as follows for particular cases in (b)</u>

Case 1: $\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}-\text{"55"}Be^{-Bt},\ -\text{"55"}Be^{-Bt}=7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: it should be – 7.5

Case 2: $\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}\text{"55"}Be^{-Bt},\ \text{"55"}Be^{-Bt}=-7.5\Rightarrow\ B=-0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 3: $\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}\text{"55"}Be^{-Bt},\ \text{"55"}Be^{-Bt}=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A0**

Error: incorrect derivative

Case 4: $\left\{\dfrac{\mathrm{d}H}{\mathrm{d}t}=\right\}-\text{"55"}Be^{-Bt},\ \text{"55"}B=7.5\Rightarrow\ B=0.136\Rightarrow H=55e^{-0.136t}+30$ scores **M1M1A1**

No errors

Question	Scheme	Marks	AOs
5(a)	$2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=...$	**M1**	1.1b
	$54-81+15+k=0 \Rightarrow k=12*$ or $-12+k=0 \Rightarrow k=12*$	**A1***	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3-9(3)^2+5(3)+12=0$	**M1**	1.1b
	$54-81+15+12=0$ Hence k = 12 *	**A1***	1.1b
		(2)	
(b)	$\int\left(2x^3-9x^2+5x+12\right)\mathrm{d}x ...x^4 \pm ...x^3 \pm ...x^2 \pm ...x \pm ...$	**M1**	3.1a
	$\frac{1}{2}(3)^4-3(3)^3+\frac{5}{2}(3)^2+12(3)+c=-10 \Rightarrow c=...$	**dM1**	1.1b
	$(0, -28)$	**A1**	2.2a
		(3)	
			(5 marks)

Notes

(a) **Mark (a) and (b) together**

M1: Substitutes x = 3 completely into the given derivative, sets = 0 and solves for k.

e.g., $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=...$

May be implied by e.g. $54-81+15+k=0 \Rightarrow k=...$ with at least 2 correctly evaluated powers.

A1*: Obtains k = 12 with no errors seen and sufficient working shown. As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow -12+k=0 \Rightarrow k=12$ **or** $54-81+15+k=0 \Rightarrow k=12$ **or** $2\times 27-9\times 9+5(3)+k=0 \Rightarrow k=12$

But $2(3)^3-9(3)^2+5(3)+k=0 \Rightarrow k=12$ scores M1A0 for lack of working

Note that some are just writing the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, then write "sub in x = 3" but don't actually show 3 substituted in and then go on to write $-12+k=0$ leading to $k=12$ scores M0A0*.

Alternative:

M1: Substitutes x = 3 and k = 12 into the given derivative and attempts to evaluate

A1*: Correct work to obtain an answer of 0 with a (minimal) conclusion e.g., tick, hence proven etc.

As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+12=54-81+15+12=0$ ✓

(b)

M1: Attempts to integrate. Evidence can be taken for integrating to obtain at least 2 from:

$2x^3 \rightarrow ...x^4$ or $-9x^2 \rightarrow ...x^3$ or $5x \rightarrow ...x^2$ or $12 \rightarrow ...x$ where … are constants

dM1: Substitutes x = 3 into their integrated expression that includes a constant of integration, sets this equal to ±10 and proceeds to find their constant. **Depends on the previous mark.**
If the substitution is not shown this mark may be implied by their value for c or by their equation e.g., $18+c=\pm 10$

A1: $(0, -28)$ Condone -28 or y = −28 but not just c = −28. There must be no other values or points.
Condone $(-28, 0)$ following y = −28

<u>Beware of circular arguments which avoid doing part (a) e.g.</u>

Integration is used on the given derivative to give y in terms of x, k and c

(3, −10) is substituted to give 3k + c = 8

Part (b) is then done first using k = 12 to find c = −28

This is then substituted into $3k + c = 8$ to give $k = 12$
This scores (a) M0A0 (b) M1dM1A1 (if –28 is identified as the intercept)

Alternative for part (a) using algebraic division:

$$
\begin{array}{r|l}
 & 2x^2 - 3x \quad - 4 \\
\hline
x-3 & 2x^3 - 9x^2 + 5x + k \\
 & \underline{2x^3 - 6x^2} \\
 & \quad -3x^2 + 5x \\
 & \quad \underline{-3x^2 + 9x} \\
 & \qquad\quad -4x + k \\
 & \qquad\quad \underline{-4x + 12} \\
 & \qquad\qquad k - 12 \text{ (or 0)}
\end{array}
$$

leading to $k - 12 = 0$ and then $k = 12$.

M1: Attempts to divide the given cubic by $(x - 3)$ and proceeds as far as a remainder set = 0. Requires at least $2x^2 \pm 3x$.

A1*: Obtains $k = 12$ with no errors seen and sufficient working. Their algebraic division needs to be correct but allow them to have either $k - 12$ or 0 as their "remainder". If their remainder is given in their working as 0 they may proceed directly to $k = 12$.

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	M1	1.1b
	$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	A1*cso	2.2a
		(2)	
(b)	Attempt to find at least two lengths between AB, BC, CD and AD $\lvert\overrightarrow{BC}\rvert = \sqrt{50^2 + 120^2} = 130$, $\lvert\overrightarrow{DA}\rvert = \sqrt{10^2 + 24^2} = 26$	M1	1.1b
	$\lvert\overrightarrow{AB}\rvert = \sqrt{12^2 + 16^2} = 20$, $\lvert\overrightarrow{CD}\rvert = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (awrt 115 m)	A1	1.1b
	Average speed $= \dfrac{2\left(26+130+20+28\sqrt{17}\right)/1000}{5/60}$	dM1	3.1b
	awrt $= 6.99$ (km/h)	A1	3.2a
		(4)	
		(6 marks)	

Notes

(a)

M1: Attempts to find both $\overrightarrow{AD} = \pm(10\mathbf{i} + 24\mathbf{j})$ and $\overrightarrow{BC} = \pm(50\mathbf{i} + 120\mathbf{j})$.

May be seen as column vectors.

Condone poor notation with column vectors e.g. $\begin{pmatrix} 10\mathbf{i} \\ 24\mathbf{j} \end{pmatrix}$ or $\begin{matrix} 10 \\ 24 \end{matrix}$ for $\begin{pmatrix} 10 \\ 24 \end{pmatrix}$

This mark can be scored for at least one correct component for each vector if no method is shown.

May be implied if they go straight for ratios (gradients) e.g. $\pm\dfrac{24-0}{22-12}$, $\pm\dfrac{16-136}{0-50}$, $\pm\dfrac{0-50}{16-136}$

Some candidates use distances in an attempt to prove part (a) e.g. finding $10^2 + 24^2$ and $50^2 + 120^2$ in which case the M1 can be implied. Adding vectors scores M0

A1*cso: This mark requires

- correct work showing AD is parallel to BC by showing that e.g. $\overrightarrow{AD} = \pm\frac{1}{5}\overrightarrow{BC}$ or equivalent e.g. $\overrightarrow{BC} = \pm 5\overrightarrow{AD}$ or e.g. $\overrightarrow{AD} = 2(5\mathbf{i} + 12\mathbf{j})$ $\overrightarrow{BC} = 10(5\mathbf{i} + 12\mathbf{j})$ or e.g. $BC = \pm 5AD$ (i.e. the vector arrows are not required) or by showing the ratio/gradient of the lines through AD and BC are equal e.g. $\dfrac{24}{10} = \dfrac{120}{50}$. Condone e.g. $\dfrac{50\mathbf{i} + 120\mathbf{j}}{10\mathbf{i} + 24\mathbf{j}} = 5$
- a (minimal) conclusion e.g. ✓, hence shown, etc. which may be in a preamble e.g. if they are parallel then $AD = kBC$…etc. If using ratios/gradients they need to say that they are parallel.
- vectors correctly calculated but allow e.g. $\overrightarrow{AD} = -10\mathbf{i} - 24\mathbf{j}$ and allow poor column vector notation as above

Using reciprocal gradients for both is acceptable for A1 even if they call them "gradients".

Do not credit work in part (b) in part (a) unless used in part (a)

(b)

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral.

May be implied by at least 2 correct lengths.

For reference $\pm\overrightarrow{AB} = \pm(-12\mathbf{i} + 16\mathbf{j})$, $\pm\overrightarrow{BC} = \pm(50\mathbf{i} + 120\mathbf{j})$, $\pm\overrightarrow{CD} = \pm(-28\mathbf{i} - 112\mathbf{j})$, $\pm\overrightarrow{DA} = \pm(10\mathbf{i} + 24\mathbf{j})$

Allow ft if using their vectors from part (a) provided subtraction was used. Do not be concerned about the signs of individual components but must be using subtraction (but condone $\pm\overrightarrow{AB} = \pm(12\mathbf{i} + 16\mathbf{j})$) to find the vectors and then squaring and adding components and then taking the square root.

A1: At least 2 lengths correct: If units are given they must be correct.

$\left|\overrightarrow{AB}\right| = \sqrt{12^2 + 16^2} = 20,$ $\left|\overrightarrow{CD}\right| = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (allow awrt 115 m)

$\left|\overrightarrow{BC}\right| = 10\sqrt{5^2 + 12^2} = 130,$ $\left|\overrightarrow{DA}\right| = 2\sqrt{5^2 + 12^2} = 26$

Allow if they are working in km e.g. $\left|\overrightarrow{AB}\right| = 0.02$ etc.

M1A1 is implied by a total distance of awrt 291 (m) or possibly a multiple of this if they are doubling (awrt 583) or e.g. multiplying by 24 (awrt 6990) etc.

dM1: For an attempt at an average speed ignoring any attempt to get the correct units.
They must have attempted all 4 lengths for this mark.
There must be some indication that they have divided by 5 but this may be implied.
Allow this mark if they calculate the average speed for 2 laps or 1 lap e.g.
$\frac{"291"\times 2}{5}$, $\frac{"291"}{5}$, $"291"\times 12$, $"291"\times 2\times 12$ or e.g. if they divide by 2.5 or multiply by 24.

A1: awrt 6.99 (km/h). or anything which truncates to 6.99 e.g. 6.995. Units are **not** required but if they are given they must be correct. Isw once a correct answer is seen.

An exact answer is acceptable for the final A1 in (b): $4.224 + 0.672\sqrt{17}$

<u>Special Case:</u>

Some candidates are misinterpreting/misreading the position vector for *B* as 16**i** rather than 16**j**
This is usually implied by their vectors/ratios e.g.

$$\overrightarrow{AD} = \pm(10\mathbf{i} + 24\mathbf{j}) \quad \text{and} \quad \overrightarrow{BC} = \pm(34\mathbf{i} + 136\mathbf{j})$$

or e.g.

$$\pm\frac{24-0}{22-12}, \ \pm\frac{50-16}{136-0}$$

For part (a), the maximum possible score is **M1A0** with the conditions for the M mark as described in the main scheme.

For part (b) the maximum possible score is **M1A1M1A0** as follows:

M1: Attempts to use Pythagoras to find **at least two** of the lengths of the quadrilateral as defined in the main scheme.

For reference $\pm\overrightarrow{AB} = \pm 4\mathbf{i}$, $\pm\overrightarrow{BC} = \pm(34\mathbf{i} + 136\mathbf{j})$, $\pm\overrightarrow{CD} = \pm(-28\mathbf{i} - 112\mathbf{j})$, $\pm\overrightarrow{DA} = \pm(10\mathbf{i} + 24\mathbf{j})$

A1: Correct lengths for *AD* and *CD*. If units are given they must be correct.
This may **not** be scored for correct ft lengths for *AB* or *BC*
So requires both:

$\left|\overrightarrow{CD}\right| = \sqrt{28^2 + 112^2} = 28\sqrt{17}$ (allow awrt 115 m)

$\left|\overrightarrow{DA}\right| = \sqrt{10^2 + 24^2} = 26$

dM1: As above for an attempt at an average speed ignoring any attempt to get the correct units.

A0: Not available

If the position vector for *B* is not misinterpreted/misread in part (b) then full marks are available.

Question	Scheme	Marks	AOs
7(a)	$x^3 \to ...x^2$ and $3y^2 \to ...y\frac{dy}{dx}$	**M1**	1.1b
	$2xy \to 2y + 2x\frac{dy}{dx}$	**B1**	1.1b
	$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$	**M1**	2.1
	$\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$	**A1**	1.1b
		(4)	
(b)	$\frac{dy}{dx} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{dy}{dx} + 2\times5 + 6\times5\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = ...\ \left(-\frac{11}{13}\right)$	**M1**	1.1b
	$y - 5 = "\frac{13}{11}"(x+2)$	**dM1**	1.1b
	$13x - 11y + 81 = 0$	**A1**	2.2a
		(3)	
			(7 marks)

Notes

(a) Allow equivalent notation for the $\frac{dy}{dx}$ e.g. y'

M1: Attempts to differentiate $x^3 \to ...x^2$ **and** $3y^2 \to ...y\frac{dy}{dx}$ where … are constants

B1: Correct application of the product rule on $2xy$: $2xy \to 2x\frac{dy}{dx} + 2y$

Note that some candidates have a spurious $\frac{dy}{dx} = ...$ at the start (as their intention to differentiate) and this can be ignored for the first 2 marks

M1: For a valid attempt to make $\frac{dy}{dx}$ the subject, with exactly 2 different terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$. Look for $(... \pm ...)\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$ which may be implied by their working.

Condone slips provided the intention is clear.

For those candidates who had a spurious $\frac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\frac{dy}{dx}$ and so score M0.

If they ignore it, then this mark is available for the condition as described above.

A1: $\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$ oe e.g. $\frac{dy}{dx} = \frac{-2y-3x^2}{2x+6y}$, $\frac{2y+3x^2}{-2x-6y}$ Isw once a correct expression is seen.

Note that it is sometimes unclear if the minus sign(s) is/are correctly placed and you may have to use your judgement. Evidence may be available in part (b) to help you decide if they have the correct expression.

(b)

M1: Substitutes x = -2 and y = 5 into $\frac{dy}{dx} = "-\frac{2y+3x^2}{2x+6y}"$

They must have x's and y's in their $\frac{dy}{dx}$ but condone slips in substitution provided the intention is clear.

As a minimum look for at least one x and at least one y substituted correctly.

Note that this mark may be implied by their value for $\frac{dy}{dx}$ and may be implied if, for example, they find the negative reciprocal or the reciprocal of $"-\frac{2y+3x^2}{2x+6y}"$ and then substitute x = -2 and y = 5

Alternatively, substitutes x = -2 and y = 5 into their attempt to differentiate and then rearranges to find a value or numerical expression for $\frac{dy}{dx}$

dM1: Attempts to find the equation of the normal using their gradient of the tangent and x = -2 and y = 5 correctly placed. Score for an expression of the form $(y-5) = "\frac{13}{11}"(x+2)$ or if they use $y = mx + c$ they must proceed as far as c =… Must be using the **negative reciprocal** of the tangent gradient.

Note that $y-5=\frac{2x+6y}{2y+3x^2}(x+2)$ is not a correct method unless the gradient is evaluated first *before* expanding.

A1: $13x-11y+81=0$ or any integer multiple of this equation including the "= 0", not just a, b, c given.

e.g., $26x-22y+162=0$ is likely if they don't cancel down their gradient.

Question	Scheme	Marks	AOs
8(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	**B1**	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\dfrac{8}{2}\Rightarrow\alpha=...$	**M1**	1.1b
	$\alpha=$ awrt 1.326	**A1**	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	**M1**	1.1b
	$9\sqrt{17}$	**A1**	2.2a
(ii)	awrt 1.33	**B1ft**	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\dfrac{8}{2}$, $\cos\alpha=\pm\dfrac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\dfrac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$ their R or allow $\pm$ 4.5R (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\dfrac{\mathrm{d}S}{\mathrm{d}x}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\dfrac{4}{\sqrt{17}},\ \cos x=\dfrac{1}{\sqrt{17}}$

$\Rightarrow 9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$ their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\text{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5\left(2\sqrt{17}\right)$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

Question	Scheme	Marks	AOs
9(a) Way 1	$x=(t+3)^2-25$	**M1**	1.1b
	$\Rightarrow x+25=(t+3)^2 \Rightarrow (x+25)^{\frac{1}{2}}=(t+3) \Rightarrow y=\ldots$	**M1**	2.1
	$y=6\ln(x+25)^{\frac{1}{2}} \Rightarrow y=3\ln(x+25)$	**A1cso**	1.1b
		(3)	
	(a) Way 2		
	$y=6\ln(t+3)=3\ln(t+3)^2$	**M1**	1.1b
	$y=3\ln(t+3)^2=3\ln(t^2+6t+9)=3\ln(x+16+9)$	**M1**	2.1
	$y=3\ln(x+25)$	**A1cso**	1.1b
	(a) Way 3		
	$y=6\ln(t+3) \Rightarrow \dfrac{y}{6}=\ln(t+3) \Rightarrow t+3=e^{\frac{y}{6}} \Rightarrow t=e^{\frac{y}{6}}-3$	**M1**	1.1b
	$x=\left(e^{\frac{y}{6}}-3\right)^2+6\left(e^{\frac{y}{6}}-3\right)-16 \Rightarrow y=\ldots$ or $x=\left(e^{\frac{y}{6}}-3+8\right)\left(e^{\frac{y}{6}}-3-2\right) \Rightarrow y=\ldots$	**M1**	2.1
	$y=3\ln(x+25)$	**A1cso**	1.1b
	(a) Way 4		
	$x=(t+3)^2-25$	**M1**	1.1b
	$\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \dfrac{3}{(t+3)^2}=\dfrac{3}{x+25} \Rightarrow y=3\ln(x+25)(+c)$	**M1**	2.1
	e.g. $t=0 \Rightarrow x=-16, y=6\ln 3 \Rightarrow 6\ln 3=3\ln(9) \Rightarrow c=0$ $y=3\ln(x+25)$	**A1cso**	1.1b
(b)	$x=0,\ y=3\ln 25$ oe e.g. $6\ln 5$	**B1ft**	2.2a
	$\dfrac{dy}{dx}=\dfrac{3}{x+"25"} \Rightarrow \dfrac{dy}{dx}=\dfrac{3}{0+"25"}\quad \left(=\dfrac{3}{25}\right)$ **or** $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \dfrac{\frac{6}{2+3}}{2\times 2+6}\quad \left(=\dfrac{6}{50}=\dfrac{3}{25}\right)$	**M1**	2.1
	$y-"3\ln 25"="\dfrac{3}{25}"(x\{-0\})$	**dM1**	3.1a
	$25y-3x=150\ln 5$	**A1**	2.2a
		(4)	
			(7 marks)

Notes

Choose the mark scheme that best matches their chosen method.

(a) **Way 1**

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm \ldots$ where $\ldots \neq 0$

M1: Rearranges their $x=(t+3)^2-25$ to either $(t+3)=\ldots$ or $(t+3)^2=\ldots$ and then substitutes correctly their expression into the parametric equation for y. So e.g., $t=\sqrt{x+25}-3 \to y=6\ln\left(\sqrt{x+25}-3\right)$ is M0.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 2

M1: Attempts to use the power rule for logarithms $y=6\ln(t+3)=\ldots\ln(t+3)^2$ where $\ldots \neq 6$

M1: Writes $y=6\ln(t+3)$ as $3\ln(t+3)^2$ and then multiplies out and substitutes correctly in for t to obtain a Cartesian equation for C

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 3

M1: Attempts to make t the subject for $y=6\ln(t+3)$ to obtain $t=\mathrm{e}^{\frac{y}{6}} \pm \ldots$ where $\ldots \neq 0$

M1: Substitutes $t=\mathrm{e}^{\mathrm{f}(y)} \pm \ldots$ correctly into $x=t^2+6t-16$ and rearranges to make y the subject.

A1cso: $y=3\ln(x+25)$ including brackets with all stages of working shown.
The "$y=$" must appear at some point.

Way 4

M1: Attempts to complete the square. Award for sight of $x=(t+3)^2 \pm \ldots$ where $\ldots \neq 0$

M1: Attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ where $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(\frac{\ldots}{t+3}\right)}{at+b}$, $a,b \neq 0$ and uses the completed square form to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and then integrates to obtain a Cartesian equation for C

A1cso: A complete method using any correct point on the curve to show that $c = 0$ and obtain $y=3\ln(x+25)$ with all stages of working shown. The "$y=$" must appear at some point.

Note that a common incorrect approach in (a) is:

$$x=t^2+6t-16=(t-2)(t+8) \Rightarrow x=t-2 \Rightarrow t=x+2 \Rightarrow y=6\ln(x+5)$$

which scores no marks.

(b)

B1ft: Deduces $y = 3\ln 25$ oe e.g $y = 6\ln 5$ but allow follow through on their Cartesian equation with $x = 0$ and apply isw after a correct value or ft value for y

M1: Attempts to find $\frac{dy}{dx}$ when $x = 0$ so score for obtaining $\frac{dy}{dx} = \frac{...}{x + "25"}$ and substituting in $x = 0$

Allow this mark if they use the letters A and B e.g. $\frac{dy}{dx} = \frac{...}{x+B} = \frac{...}{0+B}$ or allow a "made up" A and B.

or

Attempts to find $\frac{dy}{dx}$ when $t = 2$ by finding $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{\frac{6}{5}}{2\times 2+6}$ $\left(= \frac{6}{50} = \frac{3}{25}\right)$

For the derivative look for $\frac{dy}{dx} = \frac{\left(\frac{...}{t+3}\right)}{at+b}$ oe e.g. $\left(\frac{...}{t+3}\right) \times \frac{1}{at+b}$ $a, b \neq 0$

NOTE if candidates find $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} = \frac{6(2t+6)}{t+3} = 12$ we will give BOD that $t = 2$ has been used unless there is clear evidence that $t = 2$ has not been used.

dM1: Attempts to find the equation of the tangent. Score for sight of $y - "3\ln 25" = "\frac{3}{25}"(x\{-0\})$ or if they use $y = mx + c$ they must proceed as far as $c = \ldots$ **It is dependent on the previous method mark.** Must have numeric A and B now.

A1: $25y - 3x = 150\ln 5$ or any integer multiple of this equation in the form $ax + by = c\ln 5$

Question	Scheme	Marks	AOs
10(a)	e.g. $\dfrac{3kx-18}{(x+4)(x-2)} \equiv \dfrac{A}{x+4}+\dfrac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2)+B(x+4)$ **or** $\dfrac{3kx-18}{(x+4)(x-2)} \equiv \dfrac{A}{x-2}+\dfrac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4)+B(x-2)$	**B1**	1.1b
	$6k-18=6B \Rightarrow B=...$ or $-12k-18=-6A \Rightarrow A=...$ **or** $3kx-18 \equiv (A+B)x+4B-2A \Rightarrow A+B=3k,\ -18=4B-2A$ $\Rightarrow A=...$ or $B=...$	**M1**	1.1b
	$\dfrac{2k+3}{x+4}+\dfrac{k-3}{x-2}$	**A1**	1.1b
		(3)	
(b)	$\displaystyle\int \left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right) dx = ...\ln(x+4)+...\ln(x-2)$	**M1**	1.2
	$("2k+3")\ln(x+4)+("k-3")\ln(x-2)$	**A1ft**	1.1b
	$("2k+3")\ln(5)-("k-3")\ln(5) \Rightarrow ("k+6")\ln 5 = 21 \Rightarrow k=...$	**dM1**	3.1a
	$(k=)\dfrac{21}{\ln 5}-6$	**A1**	2.2a
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct form for the partial fractions and sets up the correct corresponding identity which may be implied by two equations in A and B if they are comparing coefficients.

M1: Either

- substitutes $x=2$ or $x=-4$ in an attempt to find A or B in terms of k
- expands the rhs, collects terms and compares coefficients in an attempt to find A or B in terms of k

Or may be implied by one correct fraction (numerator **and** denominator)

You may see candidates substituting two other values of x and then solving simultaneous equations.

A1: Achieves $\dfrac{2k+3}{x+4}+\dfrac{k-3}{x-2}$ with no errors. Must be the correct partial fractions not just for correct numerators. May be seen in (b). Correct answer implies B1M1A1. One correct fraction only B0M1A0

(b)

M1: Attempts to find $\int \left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right)\mathrm{d}x$. Score for either $\frac{...}{x+4}\to ...\ln(x+4)$ or $\frac{...}{x-2}\to ...\ln(x-2)$

Allow the … to be in terms of *k* or just constants but there must be no *x* terms.

Condone invisible brackets for this mark.

A1ft: $("2k+3")\ln|x+4|+("k-3")\ln|x-2|$

but condone round brackets e.g. $("2k+3")\ln(x+4)+("k-3")\ln(x-2)$ or equivalent e.g. $("2k+3")\ln(x+4)+("k-3")\ln(2-x)$

Follow through their partial fractions with numerators which must both be in terms of *k*.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of *k*. To score this mark they must have attempted to integrate their partial fractions, substituted in the correct limits, subtracted either way round, set = 21 and attempted to solve to find *k*. Condone omission of the terms containing ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

Do not be concerned with the processing as long as they proceed to *k* = …

Condone if they use *x* instead of *k* after limits have been used as long as the intention is clear.

A1: Deduces $(k=)\frac{21}{\ln 5}-6$ or exact equivalent e.g. $\frac{21-6\ln 5}{\ln 5}, \frac{21-3\ln 25}{\ln 5}$.

Allow recovery from expressions that contain e.g. ln(–5) as long as it is dealt with subsequently.

Also allow recovery from invisible brackets. Condone $x=\frac{21}{\ln 5}-6$

Some candidates may use substitution in part (b) e.g.

$$\int \left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right)\mathrm{d}x=\int \left(\frac{"2k+3"}{x+4}\right)\mathrm{d}x+\int \left(\frac{"k-3"}{x-2}\right)\mathrm{d}x$$

$$u=x+4\Rightarrow \int \left(\frac{"2k+3"}{x+4}\right)\mathrm{d}x=\int \left(\frac{"2k+3"}{u}\right)\mathrm{d}u=...\ln u$$

$$u=x-2\Rightarrow \int \left(\frac{"k-3"}{x-2}\right)\mathrm{d}x=\int \left(\frac{"k-3"}{u}\right)\mathrm{d}u=...\ln u$$

Score **M1** for integrating at least once to an appropriate form as in the main scheme e.g. …ln*u*

A1ft: For $("2k+3")\ln|u|+("k-3")\ln|u|$

but condone $("2k+3")\ln u+("k-3")\ln u$ which may be seen separately

Follow through their "*A*" and "*B*" in terms of *k*.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of *k*. To score this mark they must have attempted to integrate their partial fractions using substitution, substituted in the correct changed limits and subtracts either way round, set = 21 and attempted to solve to find k. Do not be concerned with processing as long as they proceed to $k = \ldots$ Condone omission of terms which contain e.g. ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

$$\left[(2k+3)\ln u\right]_1^5 + \left[(k-3)\ln u\right]_{-5}^{-1} = 21 \Rightarrow (2k+3)\ln 5 - (2k+3)\ln 1 + (k-3)\ln 1 - (k-3)\ln 5 = 21$$

$$\Rightarrow (2k+3)\ln 5 - (k-3)\ln 5 = 21 \Rightarrow \left(k+6\right)\ln 5 = 21 \Rightarrow k = \ldots$$

A1: $k = \dfrac{21}{\ln 5} - 6$ or exact equivalent e.g. $\dfrac{21-6\ln 5}{\ln 5}$, $\dfrac{21-3\ln 25}{\ln 5}$, $21\log_5 \mathrm{e} - 6$.

Allow recovery from expressions that contain e.g. ln(–5) as long as it is dealt with subsequently.

Also allow recovery from invisible brackets.

Question	Scheme	Marks	AOs
11(a)	$\dfrac{\mathrm{d}V}{\mathrm{d}h}=200$ oe e.g. $\dfrac{\mathrm{d}h}{\mathrm{d}V}=\dfrac{1}{200}$	**B1**	1.1b
	$\left(\dfrac{\mathrm{d}h}{\mathrm{d}t}=\right)\dfrac{\mathrm{d}h}{\mathrm{d}V}\times\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{1}{200}\times\dfrac{k}{\sqrt{h}}$	**M1**	3.1a
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\lambda}{\sqrt{h}}$ *	**A1***	2.1
		(3)	
(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow ...h^{\frac{3}{2}}=\lambda t\{+c\}$	**M1**	1.1b
	$\dfrac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\dfrac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	**A1**	1.1b
	$\dfrac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times 0+c\Rightarrow c=1.152\left(=\dfrac{144}{125}\right)$	**dM1**	3.4
	$\dfrac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times 8+\text{"}1.152\text{"}\Rightarrow\lambda=0.342\left(=\dfrac{171}{500}\right)$	**ddM1**	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\dfrac{513}{1000}t+\dfrac{216}{125}$	**A1**	3.3
		(5)	
	(b) Alternative:		
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\lambda}{\sqrt{h}}\Rightarrow\dfrac{\mathrm{d}t}{\mathrm{d}h}=\dfrac{\sqrt{h}}{\lambda}\Rightarrow t=...h^{\frac{3}{2}}(+c)$	**M1**	1.1b
	$t=\dfrac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	**A1**	1.1b
	$0=\dfrac{2(1.44)^{\frac{3}{2}}}{3\lambda}+c$ **and** $8=\dfrac{2(3.24)^{\frac{3}{2}}}{3\lambda}+c$ $\Rightarrow\lambda=...\left(\dfrac{171}{500}\right)$ **or** $c=...\left(-\dfrac{64}{19}\right)$	**dM1**	3.4
	$\Rightarrow\lambda=...\left(\dfrac{171}{500}\right)$ **and** $c=...\left(-\dfrac{64}{19}\right)$	**ddM1**	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\dfrac{513}{1000}t+\dfrac{216}{125}$	**A1**	3.3
		(5)	
(c)	$5^{\frac{3}{2}}=0.513t+1.728\Rightarrow t=...$	**M1**	3.4
	$(t=)$ awrt18.4 min	**A1**	3.2a
		(2)	
			(10 marks)

Notes

(a)

B1: For $\frac{\mathrm{d}V}{\mathrm{d}h}=200$ stated or used – may be implied by their chain rule attempt

M1: Requires:

- $\frac{\mathrm{d}V}{\mathrm{d}h}=p,\quad p>1$
- $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm\frac{k}{\sqrt{h}}$ or e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm\frac{1}{k\sqrt{h}}$ (or a suitable letter for k, which may be λ, but must **not** be a number)
- **application** of the correct chain rule $\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ or any equivalent with $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm\frac{k}{\sqrt{h}}$ or $\pm\frac{1}{k\sqrt{h}}$ and their $\frac{\mathrm{d}V}{\mathrm{d}h}$ correctly placed. So $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{200k}{\sqrt{h}}=\frac{\lambda}{\sqrt{h}}$ scores M0 as $\frac{\mathrm{d}h}{\mathrm{d}V}$ is incorrectly placed.

A1*: A rigorous argument with all steps shown and simplifies to achieve $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\lambda}{\sqrt{h}}$ with no errors.

Do not allow the use of λ for both constants. Allow use of e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=\pm\frac{1}{k\sqrt{h}}$ for full marks.

e.g. $\frac{\mathrm{d}V}{\mathrm{d}h}=200,\quad \frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{200}\times\frac{\lambda}{\sqrt{h}}=\frac{\lambda}{\sqrt{h}}$ scores B1M1A0* *unless* e.g. "let $\lambda=\frac{\lambda}{200}$" seen.

Allow correct work leading to e.g. $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{200}\times\frac{k}{\sqrt{h}}\to\frac{\lambda}{\sqrt{h}}$ or $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$ so $\lambda=\frac{k}{200}$

There must be an attempt to link the $\frac{\mathrm{d}h}{\mathrm{d}t}$ with the $\frac{\lambda}{\sqrt{h}}$ which may be missing an = sign.

Allow an argument with $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{k}{\sqrt{h}}$ e.g. $\frac{\mathrm{d}V}{\mathrm{d}h}=200,\quad \frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{200}\times-\frac{k}{\sqrt{h}}=\frac{\lambda}{\sqrt{h}}$

Withhold this A mark if there are notational errors e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=200,\quad \frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{200}\times\frac{k}{\sqrt{h}}=\frac{\lambda}{\sqrt{h}}$ scores B1(implied)M1A0*

(b) Note that some candidates may work with e.g. $\lambda=\frac{k}{200}$ or e.g. $\lambda=200k$ which is acceptable.

Candidates who do not have a λ e.g. assume $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{200\sqrt{h}}$ or e.g. $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{200}{\sqrt{h}}$ then only the first 2 method marks are available (see note below). Condone use of other variables if the intention is clear but the final answer must be in terms of *h* and *t*.

M1: Separates the variables and integrates to obtain an equation of the form $...h^{\frac{3}{2}}=\lambda t\{+c\}$ oe

The constant of integration is not needed for this mark.

A1: $\frac{2}{3}h^{\frac{3}{2}}=\lambda t(+c)$ oe. The constant of integration is not needed for this mark.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

dM1: Substitutes $t=0$ and $h=1.44$ and attempts to find c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find "c" as long as they are using $t=0$ and $h=1.44$

May be implied by their value of c.

ddM1: Substitutes $t=8$ and $h=3.24$ and their c and attempts to find λ. Do not be concerned with the "processing" to find λ as long as they are using $t=8$ and $h=3.24$.

It is dependent on both previous method marks.

A1: Correct equation in the correct form from correct work. $h^{\frac{3}{2}}=0.513t+1.728$ or $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Note candidates who do not have a λ e.g. assume $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{200\sqrt{h}}$ or e.g. $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{200}{\sqrt{h}}$ can use either $t=0$ and $h=1.44$ **or** $t=8$ and $h=3.24$ to find their constant of integration.

(b)Alternative:

M1: Finds the reciprocal of both sides and integrates to obtain an equation of the form $t=...h^{\frac{3}{2}}(+c)$

A1: $t=\dfrac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe. The constant of integration is not needed for this mark.

dM1: Substitutes $t=0$ and $h=1.44$ **and** substitutes $t=8$ and $h=3.24$ **and** attempts to find λ **or** c.

It is dependent on the previous method mark.

Do not be concerned with the "processing" to find λ or c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$ and reach a value for λ or c. May be implied by their value(s).

ddM1: Complete attempt to find λ **and** c. **It is dependent on both previous method marks.**

Do not be concerned with the "processing" to find λ and c as long as they are using $t=0$ and $h=1.44$ and $t=8$ and $h=3.24$.

A1: Correct equation in the correct form. $h^{\frac{3}{2}}=0.513t+1.728$ or $h^{\frac{3}{2}}=\dfrac{513}{1000}t+\dfrac{216}{125}$

Must follow A1 earlier so do check if this has been obtained fortuitously. Allow 1.73 for 1.728

Special Case:

Some candidates are using the given equation in part (b) to find the value of A and the value of B using the given conditions. May score a maximum of 00110. This should be marked as follows:

M0A0: (No attempt to integrate)

M1: Substitutes t = 0 and h = 1.44 to find a value for B

dM1: Substitutes t = 8 and h = 3.24 with their value of B to find a value for A

A0: Since they have not used the given model.

(Allow full recovery in (c) if this equation is correct)

(c)

M1: Attempts to substitute h = 5 into their equation which must be of the form $h^{\frac{3}{2}}=At+B$ or possibly a rearranged equation e.g. $h^{\frac{1}{2}}=\sqrt[3]{At+B}$ with values of A and B leading to a value for t.

Do not be concerned about the processing as long as they use h = 5 and obtain a value for t even if t is negative.

A1: Awrt 18.4 minutes **following a correct equation in (b)**.

The units are required but allow e.g. min, mins, but not just 18.4 and not m (which means metres)

Allow e.g. 18 minutes 25 seconds or 18 mins 26 secs

Note this may follow A0 in part (b) as they may have rearranged incorrectly in (b) but use a correct equation in (c) e.g. $\dfrac{2}{3}h^{\frac{3}{2}}=\dfrac{171}{500}t+\dfrac{144}{125}$, $h^{\frac{1}{2}}=\sqrt[3]{\dfrac{513}{1000}t+\dfrac{216}{125}}$ or may come from the special case.

Apply isw following a correct time and units, e.g., 18.4 followed by 18 mins.

Question	Scheme	Marks	AOs
12(a)	$N_A - N_B = (3+4)-(8-6) = ...$	**M1**	3.4
	5000 (subscribers)	**A1**	3.2a
		(2)	
(b)	$(T =)3$	**B1**	3.4
	This was the point when company A had the lowest number of subscribers	**B1**	2.4
		(2)	
(c)			
	$-t+7 = 2t+2$ o.e. or $t+1 = 14-2t$ o.e.	**B1**	3.1a
	$-t+7 = 2t+2$ o.e. $\Rightarrow t = ...$ or $t+1 = 14-2t$ o.e. $\Rightarrow t = ...$	**M1**	3.4
	One of the two critical values $t = \frac{5}{3}$ or $t = \frac{13}{3}$	**A1**	1.1b
	Chooses the outside region for their **two** values of t Both of $t < "\frac{5}{3}"$, $t > "\frac{13}{3}"$	**A1ft**	2.2a
	$\left\{t \in \mathbb{R} : t < \frac{5}{3}\right\} \cup \left\{t \in \mathbb{R} : t > \frac{13}{3}\right\}$	**A1**	2.5
		(5)	
(d)	The number of subscribers will become negative (when $t > 7$)	**B1**	3.5b
		(1)	
		(10 marks)	

Notes

(a)

M1: Uses the models to find the difference when $t = 0$. Allow slips in evaluating N_A and N_B but it must be clear that $t = 0$ is being used. Just 5 with no working implies M1.

A1: 5000 or 5 thousand (subscribers) (5 is A0)

(b)

B1: $(t/T =)3$ Just look for the number 3 so e.g. $t > 3$ or e.g. "just after 3" is acceptable.

If more than one value is offered then score B0 unless it is clear that the 3 is intended.
Must be seen in (b) not just on their diagram.

B1: Any acceptable reason e.g.

- This was the point when company A had the lowest number of subscribers
- After this point the number of subscribers started to increase
- It is the minimum
- Condone "it is the turning point"
- The graph changes direction
- It is the vertex
- The gradient becomes positive
- N_A increased

Allow this mark even if the first B mark was not scored e.g. $T = 3.5$ because the graph starts to increase scores B0B1

Do not allow contradictory statements.

Do not allow:

- The graph reflects at $t = 3$ on its own without further clarification

(c)

B1: Forms one valid equation (allow an equation or any inequality sign)

M1: Attempts to solve one valid equation (allow an equation or any inequality sign)

A1: For either $t=\frac{5}{3}$ or $t=\frac{13}{3}$ only (allow an equation or any inequality sign) or exact equivalent

Must be seen or used in part (c).

See notes below for attempts that use "squaring" to find the values of *t*.

A1ft: Chooses the outside region for their **two** values of t where $t > 0$.

So for $t = a$ and $t = b$ where $0 < a < b$ should be $t < a$, $t > b$. Allow , /or/and/ $\cup$ / $\cap$

Condone if incorrectly combined e.g. $"\frac{13}{3}" < t < "\frac{5}{3}"$ but **not** $"\frac{5}{3}" < t < "\frac{13}{3}"$

A1: Fully correct solution in the form $\left\{t : t < \frac{5}{3}\right\} \cup \left\{t : t > \frac{13}{3}\right\}$ or $\left\{t \middle| t < \frac{5}{3}\right\} \cup \left\{t \middle| t > \frac{13}{3}\right\}$ or

$\left(0,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ 5\right)$ either way around but condone $\left\{t < \frac{5}{3}\right\} \cup \left\{t > \frac{13}{3}\right\}$, $\left\{t : t < \frac{5}{3} \cup t > \frac{13}{3}\right\}$,

$\left\{t < \frac{5}{3} \cup t > \frac{13}{3}\right\}$ or $\left(-\infty,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ \infty\right)$.

It is not necessary to mention R , e.g. $\left\{t : t \in \mathrm{R}, t > \frac{13}{3}\right\} \cup \left\{t : t \in \mathrm{R}, t < \frac{5}{3}\right\}$

Look for $\{\ \}$ and $\cup$ or condone $\left(-\infty,\ \frac{5}{3}\right) \cup \left(\frac{13}{3},\ \infty\right)$

Do not allow solutions not in set notation such as $t < \frac{5}{3}$ or $t > \frac{13}{3}$.

Note that a lower bound for $t < \frac{5}{3}$ and an upper bound for $t > \frac{13}{3}$ are not required but may be included e.g. $\left\{t \in \mathbb{R} : 0 < t < \frac{5}{3}\right\} \cup \left\{t \in \mathbb{R} : \frac{13}{3} < t < 5\right\}$ or $\left\{t \in \mathbb{R} : 0 \leqslant t < \frac{5}{3}\right\} \cup \left\{t \in \mathbb{R} : \frac{13}{3} < t \leqslant 5\right\}$

Note that the marks in this part require **valid** equations to be solved. They must have removed the mod brackets and arrived at an equation equivalent to $-t+7=2t+2$ or $t+1=14-2t$ (all you need to check initially is whether their equation **without mod brackets** is equivalent to one of these).

Note that $\left\{t : t < \frac{5}{3},\ t > \frac{13}{3}\right\}$ is condoned for the A1ft but not for the final A1.

If x is used in their set notation then final A0, but we would condone this for the penultimate A1ft.

See notes below for answers given with no working.

(d)

B1: Requires any indication that the number of subscribers will become negative. E.g.

- It allows negative subscribers (which isn't possible)
- $8-|2t-6| \geqslant 0 \Rightarrow t \leqslant 7$ so not valid after $t = 7$ but condone not valid for t after (any value above 7)

But not

- Subscribers will become zero

Guidance for attempts that use "squaring" to find the values of *t* in (c):

Way 1:

$(-t+7)^2=(2t+2)^2$ o.e. or $(t+1)^2=(14-2t)^2$ o.e.	**B1**	3.1a
$(-t+7)^2=(2t+2)^2 \Rightarrow t=\ldots$ o.e. (Gives –9 and $\frac{5}{3}$) or $(t+1)^2=(14-2t)^2 \Rightarrow t=\ldots$ o.e. (Gives 15 and $\frac{13}{3}$)	**M1**	3.4
One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**	1.1b
Chooses the outside region for their **two** values of t Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**	2.2a
$\left\{t\in\mathbb{R}: t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}: t>\frac{13}{3}\right\}$	**A1**	2.5

Way 2:

$\lvert t-3\rvert+4=8-\lvert 2t-6\rvert \Rightarrow \lvert t-3\rvert+\lvert 2t-6\rvert=4 \Rightarrow 3t-9=4$ o.e.	**B1**	3.1a
$(3t-9)^2=4^2 \Rightarrow 9t^2-54t+81=16 \Rightarrow 9t^2-54t+65=0 \Rightarrow t=\ldots$(Gives $\frac{5}{3}$ and $\frac{13}{3}$)	**M1**	3.4
One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**	1.1b
Chooses the outside region for their **two** values of t Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**	2.2a
$\left\{t\in\mathbb{R}: t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}: t>\frac{13}{3}\right\}$	**A1**	2.5

B1: Forms one valid equation and squares both sides (allow an equation or any inequality sign)
May be implied by e.g. $(t-3+4)^2=\left(8-(2t-6)\right)^2$
Alternatively, arrives at $3t-9=4$ (o.e.) as in way 2.
M1: Attempts to solve one valid equation after squaring both sides (allow an equation or any inequality sign). Note that it is acceptable to just solve $3t-9=4$
A1: As in main scheme. **A1ft:** As in main scheme. **A1:** As in main scheme.

Note: the following is common and scores 00000.

$$\lvert t-3\rvert+4=8-\lvert 2t-6\rvert \Rightarrow (t-3)^2+4=8-(2t-6)^2$$

Which typically leads to

$$t=\frac{15\pm4\sqrt{15}}{5}$$

Guidance for answers only in part (c):

$t \ldots \text{awrt}\,1.7$ or $t \ldots \text{awrt}\,4.3$ where … is any inequality or equation scores **11000**

$t \ldots \frac{5}{3}$ or $t \ldots \frac{13}{3}$ where … is any inequality or equation scores **11100** for one correct c.v.

Both $t < \text{awrt}\,1.7$ and $t > b$ where $\left\{b > \frac{5}{3}\right\}$ scores **11010** for outside region.

Both $t < a$ and $t > \text{awrt}\,4.3$ where $\left\{a < \frac{13}{3}\right\}$ scores **11010** for outside region.

Both $t < \frac{5}{3}$ and $t > b$ where $\left\{b > \frac{5}{3}\right\}$ scores **11110** for outside region with one correct.

Both $t < a$ and $t > \frac{13}{3}$ where $\left\{a < \frac{13}{3}\right\}$ scores **11110** for outside region with one correct.

Both $t < \frac{5}{3}$ and $t > \frac{13}{3}$ scores **11110** for outside region with one correct.

Fully correct e.g. $\left\{t : t < \frac{5}{3}\right\} \cup \left\{t : t > \frac{13}{3}\right\}$ scores **11111**

Question	Scheme	Marks	AOs
13(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=3^{-2}(1+...x+...x^2)$	**M1**	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	**M1**	1.1b
	$\left(1+\frac{x}{3}\right)^{-2}=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	**A1**	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	**A1**	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1+...x+...x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k \neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1+\frac{x}{3}\right)^{-2}$ e.g. $=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2-...$ or $1-\frac{2x}{3}+\frac{x^2}{3}-...$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

<u>Direct expansion, if seen, should be marked as follows:</u>

$$\left((3+x)^{-2}=3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2\right)$$

M1: For $(3+x)^{-2}=3^{-2}+3^{-3}\times\alpha x+3^{-4}\times\beta x^2$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)\times3^{-3}x$ or $\frac{(-2)(-2-1)}{2!}\times3^{-4}x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method

Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2}\,dx$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2}\,dx$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2}\,dx$ then we will allow the M marks in (b) **only**. **M1** for $x^n \to x^{n+1}$ applied to their expansion in part (a) or 6× (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).
No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x \times "\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)"\,dx = \int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = ...$	**M1**	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	**A1**	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	**dM1**	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	**A1**	1.1b
		(4)	

(b)
M1: Attempts to multiply their expansion from part (a) by 6x or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \to x^{n+1}$ at least once having multiplied by 6x or x.
Ignore e.g. spurious integral signs.
A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18} \text{ oe e.g. } \frac{1}{9}\left(3x^2-\frac{4x^3}{3}+\frac{x^4}{2}\right),\ 6\left(\frac{x^2}{18}-\frac{2x^3}{81}+\frac{x^4}{108}\right)$$

If they have extra terms they can be ignored.
Ignore e.g. spurious integral signs.
dM1: An overall problem-solving mark for
- using part (a) by integrating $6x \times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.
This could be e.g. $\left[f(x)\right]_{0.2}^{0.4} = ...$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**
A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750} = 0.033037037...$).
Isw following a correct answer.
Note answers which use additional terms in the expansion to give a different approximation score A0
Also note that the actual value is 0.032865…

Some may use integration by parts in (b) and the following scheme should be applied.

<u>Integration by parts in (b):</u>

Either by taking $u = 6x$ and $\frac{dv}{dx} = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6\int\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right)dx$$

$$= 6x\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx \times \mathrm{f}(x) - k\int \mathrm{f}(x)dx = kx \times \mathrm{f}(x) - k\mathrm{g}(x)$$

Where f(x) is an attempt to integrate their expansion from (a) with $x^n \to x^{n+1}$ at least once **and** g(x) is an attempt to integrate their f(x) with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$ and $\frac{dv}{dx} = 6x$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right)dx$$

$$= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int\left(\frac{6x^3}{27} - \frac{6x^2}{27}\right)dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx^2 \times \mathrm{f}(x) - k\int x^2\mathrm{g}(x)dx = kx^2 \times \mathrm{f}(x) - k\mathrm{h}(x)$$

Where f(x) is their expansion from (a) and g(x) is an attempt to differentiate their f(x) with $x^n \to x^{n-1}$ at least once **and** h(x) is an attempt to integrate their $x^2\mathrm{g}(x)$ with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

(c)	Overall problem-solving mark (see notes)	**M1**	3.1a
	$u=3+x \Rightarrow \int_{3.2}^{3.4} \mathrm{f}(u)\,\mathrm{d}u \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,\mathrm{d}u = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,\mathrm{d}u \Rightarrow ...\ln u + ...u^{-1}$	**M1**	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,\mathrm{d}u = \int_{3.2}^{3.4} \frac{6}{u} - \frac{18}{u^2}\,\mathrm{d}u \Rightarrow 6\ln u + 18u^{-1}$	**A1**	1.1b
	$\left[6\ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right) = ...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	**A1**	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	**M1**	3.1a
	$\int 6x(3+x)^{-2}\,\mathrm{d}x = \frac{...x}{3+x} \pm ...\int (3+x)^{-1}\mathrm{d}x = \frac{...x}{3+x} \pm ...\ln(3+x)$ oe	**M1**	1.1b
	$= 6\ln(3+x) - \frac{6x}{3+x}$ oe	**A1**	1.1b
	$\left(6\ln(3+0.4) - \frac{6(0.4)}{3+0.4}\right) - \left(6\ln(3+0.2) - \frac{6(0.2)}{3+0.2}\right) = ...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	**A1**	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	**M1**	3.1a
	$\int 6x(3+x)^{-2}\,\mathrm{d}x = \int\left(\frac{...}{(3+x)} + \frac{...}{(3+x)^2}\right)\mathrm{d}x = ...\ln(3+x) + \frac{...}{3+x}$ oe	**M1**	1.1b
	$= 6\ln(3+x) + \frac{18}{3+x}$ oe	**A1**	1.1b
	$\left(6\ln(3+0.4) + \frac{18}{3+0.4}\right) - \left(6\ln(3+0.2) + \frac{18}{3+0.2}\right) = ...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	**A1**	2.1

(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for **<u>all of</u>**

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x} \to b\ln(3+x)$ or $\frac{a}{u} \to b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u = x+3 \to \pm\frac{a}{u} \pm b\ln u$ or e.g. $u = (x+3)^2 \to \pm\frac{a}{\sqrt{u}} \pm b\ln u$
- parts: $\pm a\ln(3+x) \pm \frac{bx}{3+x}$ condone missing brackets e.g. $...\ln x+3$ for $...\ln(3+x)$
- partial fractions: $\pm a\ln(3+x) \pm \frac{b}{3+x}$ condone missing brackets e.g. $...\ln 3+x$ for $...\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u = x+3 \rightarrow 6\ln u + 18u^{-1}$ or e.g. $u = (x+3)^2 \rightarrow 3\ln u + \dfrac{18}{\sqrt{u}}$
- parts: $6\ln(3+x) - \dfrac{6x}{3+x}$
- partial fractions: $6\ln(3+x) + \dfrac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2) + \dfrac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs.
(ignore any reference to a constant of integration)
Do not condone missing brackets e.g. $6\ln x+3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value
Depends on both previous method marks.
For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.
This could be e.g. $\left[\mathrm{f}(x)\right]_{0.2}^{0.4} = ...$ provided both previous M marks were scored.
Note that for substitution they may revert back to $3 + x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\dfrac{17}{16}\right) - \dfrac{45}{136}$ or exact equivalent e.g. $3\ln\left(\dfrac{289}{256}\right) - \dfrac{45}{136}$ or e.g. $-6\ln\left(\dfrac{16}{17}\right) - \dfrac{45}{136}$

The brackets are not required around the $\dfrac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\dfrac{1}{16}$ but not e.g. $\dfrac{3.4}{3.2}$. The $\dfrac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\dfrac{17}{16}\right| - \dfrac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.

Question	Scheme	Marks	AOs
14(a)	e.g. $2\frac{\sin\theta}{\cos\theta}(8\cos\theta+23\left(\underline{1-\cos^2\theta}\right))=8\times\underline{2\sin\theta\cos\theta\sec^2\theta}$	**B1**	1.2
	$2\tan\theta(8\cos\theta+23\sin^2\theta)=8\sin2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin2\theta$ $\sin2\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin2\theta$ $\sin2\theta(23\cos^2\theta-8\cos\theta-15)=0$	**M1A1**	2.1 2.2a
		(3)	
(b)	$\sin2x(23\cos^2x-8\cos x-15)=0$		
	$\sin2x=0\Rightarrow x=360°$ **or** $540°$	**B1**	2.2a
	$23\cos^2x-8\cos x-15\Rightarrow\cos x=-\frac{15}{23}$	**M1**	1.1b
	$\cos x=-\frac{15}{23}\Rightarrow x=...$	**dM1**	1.1b
	$x=360°, 540°$ and awrt $491°$ only	**A1**	2.3
		(4)	
		(7 marks)	

Notes

(a) Allow use of e.g. x but the final mark requires the equation to be in terms of θ

B1(M1 on EPEN): For recalling and using at least one correct trigonometric identity in the given equation.

e.g. one of: $\sin^2\theta+\cos^2\theta=1,\ 1+\tan^2\theta=\sec^2\theta,\ \tan\theta=\frac{\sin\theta}{\cos\theta},\ \sin2\theta=2\sin\theta\cos\theta$

This may be seen explicitly or may be implied by their working by e.g. $\tan\theta\cos\theta=\sin\theta$ or they might multiply both sides by $\cos^2\theta$ leaving $8\sin2\theta$ on the rhs implying $1+\tan^2\theta=\sec^2\theta$

M1: For manipulating the equation using trigonometric identities (condoning sign slips only in the identities and arithmetic slips) to obtain an expression of the form:

$A\sin2\theta\cos^2\theta+B\sin2\theta\cos\theta+C\sin2\theta\ (=0)$ or $\sin2\theta\left(A\cos^2\theta+B\cos\theta+C\right)\ (=0)$ with $A, B, C\neq 0$

A1: $\sin2\theta(23\cos^2\theta-8\cos\theta-15)=0$ oe e.g. $\sin2\theta(-23\cos^2\theta+8\cos\theta+15)=0$ cao

Note that this is not a given answer so condone notational slips e.g. $\cos\theta^2$ for $\cos^2\theta$ provided the intention is clear but the final equation must have no notational errors.

Note that the "= 0" is not required for the M1 but is required for the A1

Note: some candidates arrive at the correct final answer fortuitously following errors in their work.

(b) Allow all marks in (b) to score if the correct equation is obtained fortuitously in part (a)

Also allow use of θ instead of x throughout in part (b). Correct answers, no working scores max 1000

B1: For one of $x=360(°)$ or $x=540(°)$ Condone $x=2\pi$ or $x=3\pi$ for this mark.

The degrees symbol is not required. This may come from $\cos x = 1$

M1: Attempts to solve their 3TQ from part (a) or a "made up" 3TQ (which may only be seen in (b)) leading to a value for $\cos x$. The general guidance for solving a 3 term quadratic equation can be applied. Allow solution(s) from a calculator which may be implied by at least one correct value for their 3TQ. Must be a value for $\cos x$ and not e.g. x.

dM1: Attempts to find one of their angles in the range $360<x<540$ (but not 450) for their $\cos x = k$ where $|k|<1$ May be implied by their value(s) but must be in degrees.

Requires them to state a value for $\cos x$. Must be checked (you can check cos(their x) = their k (1sf))

A1: $x=360°, 540°$ and awrt $491°$ only with no other values in range (including 450).

The degrees symbol is not required. awrt 491 must come from $\cos x=-\frac{15}{23}$

Question	Scheme	Marks	AOs
15	$(\sin x-\cos x)^2<1\Rightarrow \sin^2 x-2\sin x\cos x+\cos^2 x\ \ (<1)$ o.e.	**M1**	1.1b
	Examples: $1-2\sin x\cos x<1,\ 1-\sin 2x<1,\ -2\sin x\cos x<0,\ -\sin 2x<0$	**A1**	2.2a
	As x is obtuse then $-2\sin x\cos x$ is positive because $\sin x>0$ and $\cos x<0$ so we have a contradiction. Therefore $\sin x-\cos x \ldots 1$ *	**A1***	2.4
		(3 marks)	

Notes

Condone poor notation e.g. $\sin x^2$ **or e.g.** $-2\sin\theta\cos x<1$ **for the first two marks only.**

M1: Expands $(\sin x-\cos x)^2$ to obtain $\sin^2 x\pm k\sin x\cos x+\cos^2 x$ where k = 1 or 2 o.e. May be implied.

A1: Uses a correct identity $\sin^2 x+\cos^2 x=1$ or e.g. $-\sin^2 x-\cos^2 x=-1$ to obtain a correct inequality in any form that does not include the $\sin^2 x$ and $\cos^2 x$ terms. Condone e.g. $-2\sin\ \cos x<0$

A1*: Fully correct work which includes

- a convincing argument that explains why their inequality is not true
- a statement that indicates there is a contradiction
- a conclusion that $\sin x-\cos x \ldots 1$ (there is no need to repeat "when x is obtuse")
- no contradictory statements
- no mixed/missed variables, e.g., $-2\sin\theta\cos x<1$ or $1-\sin 2<1$

Examples:

From $-2\sin x\cos x<0$:

In the second quadrant $-2\sin x\cos x$ is $-\times+\times-=+$

"(this is a) contradiction" or equivalent (therefore) $\underline{\sin x-\cos x \ldots 1}$

or

As x is obtuse, $\sin x$ > 0, $\cos x$ < 0 so $-2\sin x\cos x>0$

"(this is a) contradiction" or equivalent (therefore) $\underline{\sin x-\cos x \ldots 1}$

From $-\sin 2x<0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. $\pi<2x<2\pi$) so $-\sin 2x>0$

"(this is) wrong" or equivalent (therefore) $\underline{\sin x-\cos x \ldots 1}$

From $1-\sin 2x<1$:

As x is obtuse, $2x$ is reflex o.e. (i.e. 180 < $2x$ < 360) so $\sin 2x<0$ so $1-\sin 2x>1$

"(this is a) contradiction" or equivalent (therefore) $\underline{\sin x-\cos x \ldots 1}$

From $\sin 2x>0$:

As x is obtuse, $2x$ is reflex o.e. (i.e. 180 < $2x$ < 360) so $\sin 2x<0$

"(this is) incorrect" or equivalent (therefore) $\underline{\sin x-\cos x \ldots 1}$

Note that you may condone the absence of a statement referring to the fact that $(\sin x-\cos x)^2<1$ is only valid since $\sin x-\cos x>0$ when x is obtuse.

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