

MathsWithDan – Factorising and Solving Quadratics

Starter

1. Expand

$$6a(a - 4)$$

(1 mark)

2. Factorise fully

$$5y + 20$$

(1 mark)

3. Expand and simplify

$$(x + 5)(x - 2)$$

(2 marks)

4. Expand and simplify

$$(m - 4)^2$$

(2 marks)

(Total 6 marks)



Identifying Quadratic Equations

For each equation, state whether it is quadratic or not quadratic.

1.

$$x^2 + 7x - 18 = 0$$

☐ Quadratic ☐ Not quadratic

2.

$$x^3 + 2x^2 - 5x = 12$$

☐ Quadratic ☐ Not quadratic

3.

$$4x^2 - 20 = 0$$

☐ Quadratic ☐ Not quadratic

4.

$$(2x - 3)(x + 5) = 0$$

☐ Quadratic ☐ Not quadratic

5.

$$6x - 11 = 0$$

☐ Quadratic ☐ Not quadratic



Quadratic Equations

Before solving an equation, it is important to identify what type of equation you are dealing with.

This is because different types of equations require different methods to solve them.

For example,

$$3x + 7 = 19$$

is a linear equation, whereas

$$x^2 + 7x + 10 = 0$$

is a quadratic equation.

A quadratic equation is an equation in which the highest power of the variable is 2.

Quadratic equations can be written in the form

$$ax^2 + bx + c = 0$$

where a , b and c are constants and $a \neq 0$.

Solving Quadratic Equations by Factorising

There are different methods that can be used to solve quadratic equations.

In this lesson, we will focus on solving quadratic equations by factorising.

Consider

$$x^2 + 7x + 10 = 0$$

Step 1 — Factorise the quadratic

We need two numbers that multiply to give 10 and add to give 7.

These numbers are 5 and 2.



Therefore,

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

so

$$(x + 5)(x + 2) = 0$$

Step 2 — Set each factor equal to zero

For the product of two factors to equal zero, at least one of the factors must equal zero.

Therefore,

$$x + 5 = 0$$

or

$$x + 2 = 0$$

Step 3 — Solve each equation

$$x = -5$$

or

$$x = -2$$

Therefore, the solutions are

$$\boxed{x = -5 \text{ or } x = -2}$$

Key Steps

To solve a quadratic equation by factorising:

1. Rearrange the equation so that it is equal to 0, if necessary.
2. Factorise the quadratic expression.



3. Set each factor equal to 0.

4. Solve each resulting equation.

Note: Not all quadratic equations can be factorised. This does not mean that they cannot be solved. We will learn other methods for solving quadratic equations later in the course. For now, we will focus on solving quadratics that can be factorised.

Worked Example 1

Solve by factorising

$$x^2 + 9x + 20 = 0$$



Worked Example 2

b) Solve by factorising

$$x^2 - 3x - 18 = 0$$



Moderate Questions

1. Factorise

$$x^2 + 11x + 24$$

(2 marks)

2. Factorise

$$x^2 + 3x - 28$$

(2 marks)

3. Solve

$$x^2 - 10x + 21 = 0$$

(3 marks)



4. Solve

$$x^2 - 5x - 24 = 0$$

(3 marks)

5. Solve

$$x^2 - 13x + 36 = 0$$

(3 marks)

(Total 13 marks)



Difference of Two Squares

A difference of two squares is a special type of quadratic expression.

It has the form

$$a^2 - b^2$$

and factorises using the rule

$$a^2 - b^2 = (a + b)(a - b)$$

Notice that:

- both terms are perfect squares
- the terms are separated by a minus sign
- the factorised form contains the same two terms, with one + and one –

For example,

$$x^2 - 25$$

is a difference of two squares because

$$x^2 - 25 = x^2 - 5^2$$

Here, $a = x$ and $b = 5$, so therefore:

$$x^2 - 25 = x^2 - 5^2 = (x + 5)(x - 5)$$



Worked Example 3

Solve

$$x^2 - 36 = 0$$



★ Exam Challenge 1

Solve

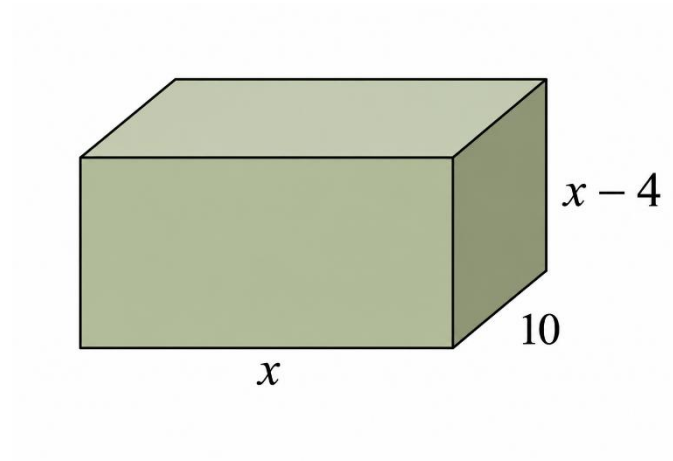
$$\frac{24}{x+2} = x+4$$

4 marks

★ Exam Challenge 2

The diagram shows a cuboid.

The volume of the cuboid is 600 cm^3 .



(a) Show that

$$x^2 - 4x - 60 = 0$$

(3 marks)



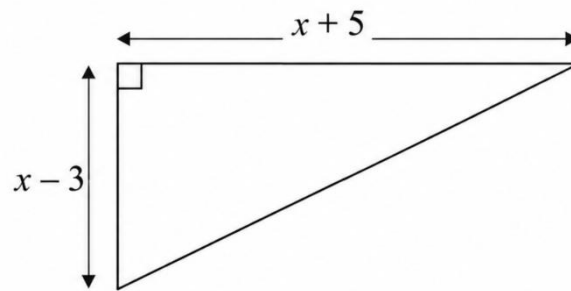
(b) Hence find x , the length of the cuboid.

Give your answer in centimetres.

(3 marks)

★ Exam Challenge 3

The diagram shows a right-angled triangle.



All the measurements are in centimetres.

The area of the triangle is 24 cm^2 .

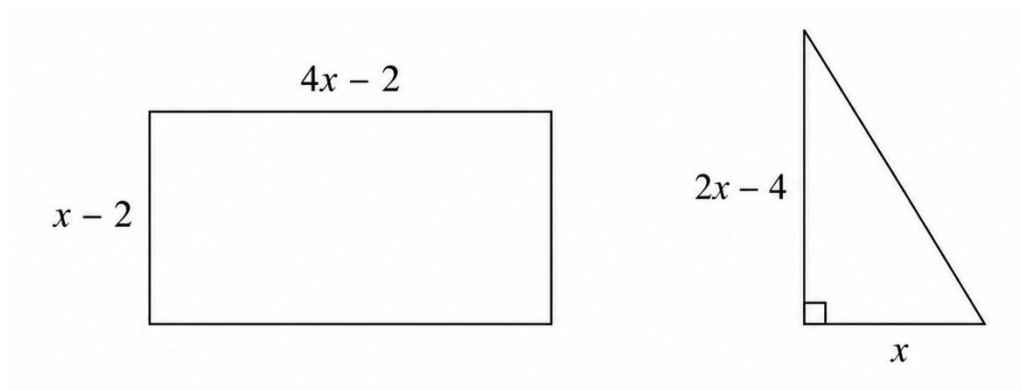
Work out the length of the shortest side of the triangle.
You must show all your working.

(4 marks)



★ Exam Challenge 4

Here is a rectangle and a right-angled triangle.



All measurements are in centimetres.

The area of the rectangle is greater than the area of the triangle.

Show that

$$3x^2 - 8x + 4 > 0$$

(3 marks)



WOW, DONE ALREADY?! THAT WAS QUICK! SEE YOU SOON!



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