



MathsWithDan – Factorising and Solving Quadratics

Starter

Starter — Answers

1. Expand

$$6a(a - 4) = \boxed{6a^2 - 24a}$$

(1 mark)

2. Factorise fully

$$5y + 20 = \boxed{5(y + 4)}$$

(1 mark)

3. Expand and simplify

$$\begin{aligned} (x + 5)(x - 2) \\ = x^2 - 2x + 5x - 10 \\ \boxed{x^2 + 3x - 10} \end{aligned}$$

(2 marks)

4. Expand and simplify

$$\begin{aligned} (m - 4)^2 \\ = (m - 4)(m - 4) \\ = m^2 - 4m - 4m + 16 \\ \boxed{m^2 - 8m + 16} \end{aligned}$$

(2 marks)

Total: 6 marks



Identifying Quadratic Equations — Mark Scheme

For each equation, state whether it is quadratic or not quadratic.

1.

$$x^2 + 7x - 18 = 0$$

☒ Quadratic ☐ Not quadratic

Reason: The highest power of x is 2.

[1 mark]

2.

$$x^3 + 2x^2 - 5x = 12$$

☐ Quadratic ☒ Not quadratic

Reason: The equation contains an x^3 term, so the highest power of x is 3.

[1 mark]

3.

$$4x^2 - 20 = 0$$

☒ Quadratic ☐ Not quadratic

Reason: The highest power of x is 2. A quadratic does not need to contain an x term.

[1 mark]



4.

$$(2x - 3)(x + 5) = 0$$

☒ Quadratic ☐ Not quadratic

Reason: Expanding the brackets gives

$$\begin{aligned} 2x^2 + 10x - 3x - 15 &= 0 \\ 2x^2 + 7x - 15 &= 0 \end{aligned}$$

so the highest power of x is 2.

[1 mark]

5.

$$6x - 11 = 0$$

☐ Quadratic ☒ Not quadratic

Reason: The highest power of x is 1, so this is a linear equation.

[1 mark]

Total: 5 marks



Worked Example 1

Solve by factorising

$$x^2 + 9x + 20 = 0$$

Working

$$x^2 + 9x + 20 = 0$$

We need two numbers that multiply to give 20 and add to give 9. These numbers are 5 and 4, so factorise the quadratic expression.

$$(x + 5)(x + 4) = 0$$

Set each factor equal to zero.

$$x + 5 = 0 \quad \text{or} \quad x + 4 = 0$$

Solve each equation.

$$x = -5 \quad \text{or} \quad x = -4$$

Therefore, the solutions are $x = -5$ or $x = -4$.

Answer: $x = -5$ or $x = -4$



Worked Example 2

b) Solve by factorising

$$x^2 - 3x - 18 = 0$$

Working

$$x^2 - 3x - 18 = 0$$

We need two numbers that multiply to give -18 and add to give -3 . These numbers are 6 and -3 , so factorise the quadratic expression.

$$(x - 6)(x + 3) = 0$$

Set each factor equal to zero.

$$x - 6 = 0 \quad \text{or} \quad x + 3 = 0$$

Solve each equation.

$$x = 6 \quad \text{or} \quad x = -3$$

Therefore, the solutions are $x = 6$ or $x = -3$.

Answer: $x = 6$ or $x = -3$



Moderate Questions

Moderate Questions — Mark Scheme

1. Factorise

$$\begin{aligned}x^2 + 11x + 24 \\ = (x + 3)(x + 8)\end{aligned}$$

M1 Identifies two numbers that multiply to 24 and add to 11.

A1 $(x + 3)(x + 8)$

(2 marks)

2. Factorise

$$\begin{aligned}x^2 + 3x - 28 \\ = (x + 7)(x - 4)\end{aligned}$$

M1 Identifies two numbers that multiply to -28 and add to 3.

A1 $(x + 7)(x - 4)$

(2 marks)

3. Solve

$$\begin{aligned}x^2 - 10x + 21 &= 0 \\ (x - 3)(x - 7) &= 0 \\ x - 3 &= 0 \text{ or } x - 7 = 0 \\ \boxed{x = 3 \text{ or } x = 7}\end{aligned}$$

M1 Correct factorisation.

M1 Sets each factor equal to zero / attempts to solve.

A1 $x = 3, x = 7$

(3 marks)



4. Solve

$$\begin{aligned}x^2 - 5x - 24 &= 0 \\(x - 8)(x + 3) &= 0 \\x - 8 &= 0 \text{ or } x + 3 = 0 \\x &= 8 \text{ or } x = -3\end{aligned}$$

M1 Correct factorisation.

M1 Sets each factor equal to zero / attempts to solve.

A1 $x = 8, x = -3$

(3 marks)

5. Solve

$$\begin{aligned}x^2 - 13x + 36 &= 0 \\(x - 4)(x - 9) &= 0 \\x - 4 &= 0 \text{ or } x - 9 = 0 \\x &= 4 \text{ or } x = 9\end{aligned}$$

M1 Correct factorisation.

M1 Sets each factor equal to zero / attempts to solve.

A1 $x = 4, x = 9$

(3 marks)

Total: 13 marks



Worked Example 3

Solve

$$x^2 - 36 = 0$$

Working (difference of two squares)

$$x^2 - 36 = 0$$

Recognise 36 as a perfect square ($36 = 6^2$).

$$x^2 - 6^2 = 0$$

Write the expression as a difference of two squares.

$$(x + 6)(x - 6) = 0$$

Use $a^2 - b^2 = (a + b)(a - b)$.

$$x + 6 = 0 \quad \text{or} \quad x - 6 = 0$$

Set each factor equal to zero.

$$x = -6 \quad \text{or} \quad x = 6$$

Therefore, the solutions are $x = -6$ or $x = 6$.

Alternative method (moving 36 to the right-hand side)

$$x^2 - 36 = 0$$

Add 36 to both sides.

$$x^2 = 36$$

Take the square root of both sides.

$$x = \pm\sqrt{36}$$

Remember to include both the positive and negative square roots.

$$x = \pm 6$$

$$x = -6 \quad \text{or} \quad x = 6$$

Therefore, the solutions are $x = -6$ or $x = 6$.

Answer: $x = -6$ or $x = 6$



★ Exam Challenge 1

Solve

$$\frac{24}{x+2} = x+4$$

Multiply both sides by $x+2$:

$$24 = (x+4)(x+2)$$

M1 — Multiplies through by $x+2$ correctly.

Expand:

$$24 = x^2 + 6x + 8$$

Rearrange to form a quadratic equation:

$$x^2 + 6x - 16 = 0$$

M1 — Expands and rearranges correctly.

Factorise:

$$(x+8)(x-2) = 0$$

A1 — Correct factorisation.

Therefore,

$$\boxed{x = -8 \text{ or } x = 2}$$

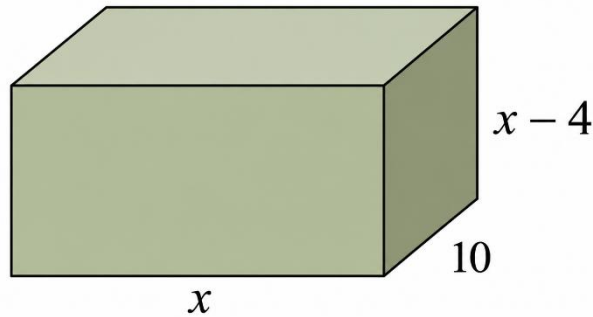
A1 — Both correct solutions.

(Total 4 marks)

★ Exam Challenge 2

The diagram shows a cuboid.

The volume of the cuboid is 600 cm^3 .



(a) Show that

$$x^2 - 4x - 60 = 0$$

(3 marks)

Volume of cuboid:

$$x \times 10 \times (x - 4) = 600$$

M1 — Forms the correct equation using the volume of the cuboid.

$$10x(x - 4) = 600$$

Expand:

$$10x^2 - 40x = 600$$

M1 — Correctly expands the brackets.

Divide by 10 and rearrange:

$$\begin{array}{l} x^2 - 4x = 60 \\ \boxed{x^2 - 4x - 60 = 0} \end{array}$$

A1 — Obtains the required quadratic equation.

(3 marks)



(b) Hence find x , the length of the cuboid.

Give your answer in centimetres.

(3 marks)

From part (a),

$$x^2 - 4x - 60 = 0$$

Factorise:

$$(x - 10)(x + 6) = 0$$

M1 — Correct factorisation.

Therefore,

$$x = 10 \text{ or } x = -6$$

Since x represents a length, $x = -6$ is not possible.

$$\boxed{x = 10 \text{ cm}}$$

A1 — Obtains the positive solution $x = 10$.

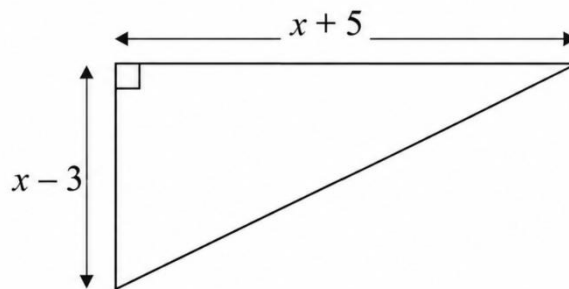
A1 — Rejects the negative solution and gives the answer with units.

(3 marks)

Total: 6 marks

★ Exam Challenge 3

The diagram shows a right-angled triangle.



All the measurements are in centimetres.

The area of the triangle is 24 cm^2 .

Work out the length of the shortest side of the triangle.
You must show all your working.

(4 marks)

Area of triangle:

$$\begin{aligned}\frac{1}{2}(x-3)(x+5) &= 24 \\ (x-3)(x+5) &= 48\end{aligned}$$

M1 — Forms the correct equation using the area of the triangle.

Expand:

$$\begin{aligned}x^2 + 2x - 15 &= 48 \\ x^2 + 2x - 63 &= 0\end{aligned}$$

M1 — Expands and rearranges to form a quadratic equation.

Factorise:

$$\begin{aligned}(x+9)(x-7) &= 0 \\ x &= -9 \text{ or } x = 7\end{aligned}$$



Reject $x = -9$, since the lengths of the triangle must be positive.

M1 — Correctly solves the quadratic and selects the valid value $x = 7$.

The two perpendicular sides are:

$$\begin{aligned}x - 3 &= 7 - 3 = 4 \\x + 5 &= 7 + 5 = 12\end{aligned}$$

Therefore, the shortest side is

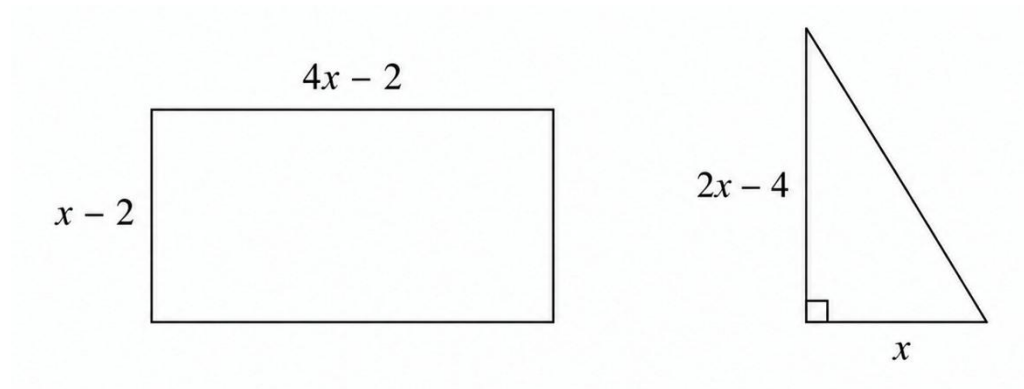
4 cm

A1 — Gives the correct length of the shortest side.

(4 marks)

★ Exam Challenge 4

Here is a rectangle and a right-angled triangle.



All measurements are in centimetres.

The area of the rectangle is greater than the area of the triangle.

Show that

$$3x^2 - 8x + 4 > 0$$

(3 marks)

Area of the rectangle:

$$(4x - 2)(x - 2)$$

Expand:

$$\begin{aligned} &4x^2 - 8x - 2x + 4 \\ &= 4x^2 - 10x + 4 \end{aligned}$$

M1 — Correct expression for the area of the rectangle and expansion.

Area of the right-angled triangle:

$$\begin{aligned} &\frac{1}{2} \times x \times (2x - 4) \\ &= x^2 - 2x \end{aligned}$$

M1 — Correct expression for the area of the triangle.

Since the area of the rectangle is greater than the area of the triangle,



$$4x^2 - 10x + 4 > x^2 - 2x$$

Rearrange:

$$3x^2 - 8x + 4 > 0$$

Therefore,

$$\boxed{3x^2 - 8x + 4 > 0}$$

A1 — Obtains the required inequality.

(3 marks)

WOW, DONE ALREADY?! THAT WAS QUICK! SEE YOU SOON!



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