



MathsWithDan – Differentiation – The Chain Rule

Starter

Differentiate:

1. $y = 4x^3 + x^2 + 5x + 1$ [2 Marks]

2. $y = (2x + 1)^3$ [3 Marks]



Composite Functions (2 Marks)

It is given that

$$f(x) = x^3 \quad \text{and} \quad g(x) = x + 1$$

Find

$$f(g(x)).$$

Challenge

How would you differentiate

$$y = (2x + 1)^8?$$

Have a think about it before we begin the lesson.

The Chain Rule

Key Idea

The Chain Rule is used to differentiate composite functions (also known as functions of another function).

A composite function is created when one function is inside another.

For example:

$$(2x + 1)^5, \sin(3x), (x^2 + 4)^7$$

In each case, there is an outer function and an inner function.

The Chain Rule

If

$$y = f(u)$$

and

$$u = g(x),$$

then

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$

where:

- $\frac{dy}{du}$ is the derivative of the outer function.
 - $\frac{du}{dx}$ is the derivative of the inner function.
-



Key Steps

When differentiating using the Chain Rule:

1. Identify the inner function.
2. Differentiate the outer function, keeping the inside unchanged.
3. Multiply by the derivative of the inner function.
4. Simplify your final answer where appropriate.



Worked Example 1

Given that $y = (2x^3 - 5x)^6$, find $\frac{dy}{dx}$ using the Chain Rule.



Worked Example 2

Given that $y = \cos^4(x)$, find $\frac{dy}{dx}$ using the Chain Rule.



You can write the Chain Rule using function notation:

- The Chain Rule enables you to differentiate a function of a function. In general,

- if $y = (f(x))^n$ then $\frac{dy}{dx} = n(f(x))^{n-1} f'(x)$

- if $y = f(g(x))$ then $\frac{dy}{dx} = f'(g(x)) g'(x)$



Worked Example 3

Given that $y = \sqrt{3x^2 + 7}$, find $\frac{dy}{dx}$ at $(2, \sqrt{19})$.



Moderate Questions

a) Differentiate

$$y = (3x - 2)^5$$

b) Differentiate

$$y = \frac{1}{5 - 4x}$$



c) Differentiate

$$y = \sqrt{2x + 9}$$



The following particular case of the Chain Rule is useful for differentiating functions that are not in the form $y = f(x)$.

- $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

Hint: This is because

$$\frac{dy}{dx} \times \frac{dx}{dy} = \frac{dy}{dy} = 1.$$

Worked Example 4

Find the value of $\frac{dy}{dx}$ at the point $(3,1)$ on the curve with equation $y^3 + 2y = x$.



★ Exam Challenge

The curve C has equation

$$y = (x^2 - 4)^3.$$

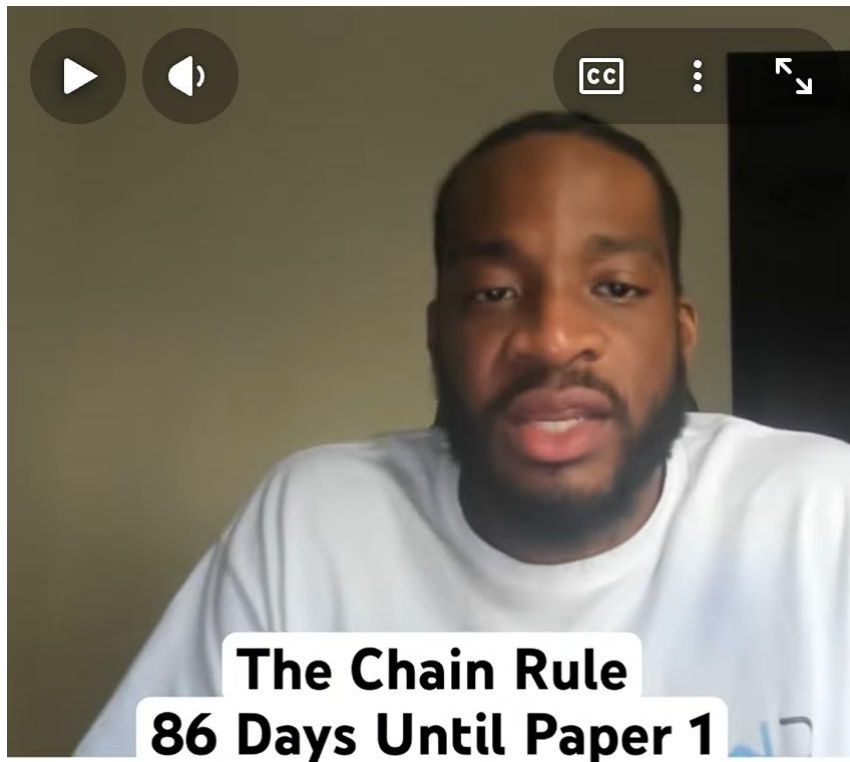
Use calculus to determine the set of values of x for which the curve is increasing.

Give your answer in set notation.

(7 marks)



 Watch Me Solve It



Use the chain rule to find $\frac{dy}{dx}$ when $y = (x^3 + 5x)^7$

[2 marks]



 Try it first on the next page and then Scan the QR code to watch the full solution.



Use the chain rule to find $\frac{dy}{dx}$ when $y = (x^3 + 5x)^7$. [2 marks]

WOW, DONE ALREADY?! THAT WAS QUICK! SEE YOU SOON!



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